EXAMPLE PROBLEMS

Experiment 1: Toss a coin and observe the up face.  
Sample Space:  1. Observe a head  
               2. Observe a tail  
This sample space can be represented in set notation as a set containing two simple events  

S = \{H, T\}

Where H represents the simple event Observe a head and T represents the simple event Observe a tail.

Venn diagram:

Experiment 2: Toss a die and observe the up face.  
Sample space:  1. Observe a 1  
               2. Observe a 2  
               3. Observe a 3  
               4. Observe a 4  
               5. Observe a 5  
               6. Observe a 6  
This simple space can be represented in set notation as a set of six simple events  

S = \{1, 2, 3, 4, 5, 6\}

Venn diagram:
Sample spaces with large or infinite number of sample points are best described by a statement or rule.

**Example:** If the possible outcomes of an experiment are the set of cities in the world with a population over 1 million, our sample space is written as

\[ S = \{x \mid x \text{ is a city with a population over } 1 \text{ million}\} \]

which reads “S is the set of all x such that x is a city with a population over 1 million.”

**Example: Event**

Given the sample space \( S = \{ t \mid t \geq 0 \} \), where t is the life in years of a certain electronic component, then the event \( A \) that the components fails before the end of the fifth year is the subset \( A = \{t \mid 0 < t < 5\} \).
3.5/66 M/S) The YES/MVS (Yorktown Expert System/MVS Manager) is an experimental expert system designed to exert active control over a computer system and provide advice to computer operators. YES/MVS is designed with a knowledge base consisting of 548 rules that are triggered in response to messages or queries from the computer operator. The accompanying table gives the number of rules allocated to different subdomains of the operator’s actions. Periodically, the rules in the YES/MVS knowledge base are tested and adjusted, if necessary. Suppose a rule is selected at random for testing and its type (operator action/query) noted.

<table>
<thead>
<tr>
<th>OPERATOR ACTION/QUERY</th>
<th>NUMBER OF RULES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batch scheduling</td>
<td>139</td>
</tr>
<tr>
<td>JES queue space</td>
<td>104</td>
</tr>
<tr>
<td>C-to-C links</td>
<td>68</td>
</tr>
<tr>
<td>Hardware error</td>
<td>87</td>
</tr>
<tr>
<td>SMF management</td>
<td>25</td>
</tr>
<tr>
<td>Quiesce and IPL</td>
<td>52</td>
</tr>
<tr>
<td>Performance</td>
<td>41</td>
</tr>
<tr>
<td>Background monitor</td>
<td>32</td>
</tr>
<tr>
<td>TOTAL</td>
<td>548</td>
</tr>
</tbody>
</table>

a. List the simple events for this experiment.
b. Assign probabilities to the simple events based on the information contained in the table.
c. What is the probability the rule is a C-to-C link or hardware error rule?
d. What is the probability the rule is not a performance rule?

Solution:
a. The simple events are the eight action/queries listed in the table.
b. Probabilities

<table>
<thead>
<tr>
<th>OPERATOR ACTION/QUERY</th>
<th>NUMBER OF RULES</th>
<th>PROBABILITIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batch scheduling</td>
<td>139</td>
<td>139/548</td>
</tr>
<tr>
<td>JES queue space</td>
<td>104</td>
<td>104/548</td>
</tr>
<tr>
<td>C-to-C links</td>
<td>68</td>
<td>68/548</td>
</tr>
<tr>
<td>Hardware error</td>
<td>87</td>
<td>87/548</td>
</tr>
<tr>
<td>SMF management</td>
<td>25</td>
<td>25/548</td>
</tr>
<tr>
<td>Quiesce and IPL</td>
<td>52</td>
<td>52/548</td>
</tr>
<tr>
<td>Performance</td>
<td>41</td>
<td>41/548</td>
</tr>
<tr>
<td>Background monitor</td>
<td>32</td>
<td>32/548</td>
</tr>
<tr>
<td>TOTAL</td>
<td>548</td>
<td>548/548 = 1</td>
</tr>
</tbody>
</table>

c. \( P(\text{C-to-C link or hardware error rule}) = (68 + 87)/548 = 155/548 \)
d. \( P(\text{Not a performance rule}) = 1 – P(\text{performance rule}) = 1 – 41/548 = 507/548 \)
**Counting Rules**

**Example:** A product (e.g. hardware for a computer system) can be shipped by four different airlines and each airline can ship via three different routes. How many distinct ways exist to ship the product?

**Decision tree:** Pictorial representation of the different ways to ship the product

![Decision Tree Diagram]

The decision tree clearly shows that there are \(4 \times 3 = 12\) distinct ways to ship the product.
Example:
A company specializing in data-communications hardware markets a computing system with two types of hard disk drives, four types of display station, and two types of interfacing. How many systems would the company have to distribute if it received one order for each possible combination of hard disk drive, display station, and interfacing?

Decision tree:
Example: Multiplicative rule

A company specializing in data-communications hardware markets a computing system with two types of hard disk drives, four types of display stations, and two types of interfacing. How many systems would the company have to distribute if it received one order for each possible combination of hard disk drive display station, and interfacing?

**Soln:**

- k sets of elements, n1 in the first set, n2 in the second set and n3 in the third set, and form a sample of k elements by taking one element from each of the k sets

Let

- n1 = the types of hard disk
- n2 = the types of display stations
- n3 = the types of interfacing

Hence the no. of possible combinations is equal to

\[ n1 \times n2 \times n3 = 2 \times 4 \times 2 = 16 \]

7/24) A developer of a new subdivision offers a prospective home buyer a choice of 4 designs, 3 different heating systems, a garage or carport, and a patio or screened porch. How many different plans are available to this buyer?

**Sol’n:**

- k sets of elements, n1 in the first set, n2 in the second set, n3 in the third set, n4 in the forth set, and form a sample of k elements by taking one element from each of the k sets

\[ n1 \times n2 \times n3 \times n4 = 4 \times 3 \times 2 \times 2 = 48 \]
Example Problems: Probability

Example: Permutation rule
13/24) A witness to a hit-and-run accident told the police that the license number contained the letters RLH followed by three digits, the first of which was a five. If the witness cannot recall the last two digits, but is certain that all three digits are different, find the maximum number of automobile registrations that the police may have to check.

Sol’n:
- N distinct elements, select \( r \) elements from the N and arrange them within \( r \) positions

\[
^{N}P_r = \frac{N!}{(N-r)!} = \frac{9!}{9-2)!} = 9 \times 8 = 72
\]

18/24) Four married couples have bought 8 seats in a row for a concert. In how many different ways can they be seated

a) with no restrictions? Soln: \( n! = 8! = 40,320 \)
b) if each couple is to sit together? Soln: \( n!*n1!*n2!*n3!*n4! = 4!*2!*2!*2!*2! = 384 \)
c) if all the men sit together to the right of all the women? Soln: \( n1!*n2! = 4!*4! = 576 \)

24/25) In how many ways can a caravan of 8 covered wagons be arranged in a circle?

Sol’n: \( (n - 1)! = (8 - 1)! = 5,040 \)
Example: Combinations rule
3.10/77)(M/S) Suppose you need to replace 5 gaskets in a nuclear-powered device. If you have a box of 20 gaskets from which to make the selection, how many different choices are possible; i.e., how many different samples of 5 gaskets can be selected from 20?

Sol’n:
- r elements chosen from a set of N elements (order is not important)

\[
\binom{N}{r} = \frac{N!}{r!(N-r)!} = \frac{20!}{5!(20-5)!} = 15,504
\]

Example: Partitions rule
In order to evaluate the traffic control systems of four facilities relying on computer-based equipment, the Federal Aviation Administration (FAA) formed a 16-member task force. If the FAA wants to assign 4 task force members to each facility, how many different assignments are possible?

Sol’n:
- N distinct elements, partitioned into k sets

\[
\frac{N!}{n_1!n_2!n_3!n_4!} = \frac{16!}{4!4!4!} = 63,063,000
\]
Example: Conditional probability
The probability that a data-communications system will have high selectivity is .82, the probability that it will have high fidelity is .59, and the probability that it will have both is .33. Find the probability that a system with high fidelity will also have high selectivity.

Sol’n:
Let $S$ is the event that a data-communications system will have high selectivity

$F$ is the event that a data-communications system will have high fidelity

Probabilities:

\[
P(S) = .82 \\
P(F) = .59 \\
P(F \cap S) = .33 \\
\]

\[
P(S/F) = \frac{P(F \cap S)}{P(F)} = \frac{.33}{.59} = .5593
\]

Experience has shown that a manufacturer of computer software produces, on the average, only 1 defective blank diskette in 100. Of the next three blank diskettes manufactured, what is the probability that all three will be nondefective?

Let $D$ = the event that a defective blank diskette is produced

$ND$ = the event that a nondefective blank diskette is produced

\[
P(D) = 1/100 \\
P(ND) = 99/100
\]

\[
P(\text{three blank diskettes to be ND}) = .99(.99)(.99) = .97
\]
**Example: Conditional probability**

3.26/88) A survey of users of word processors showed that 10% were dissatisfied with the word-processing system they are currently using. Half of those who were dissatisfied had purchased their systems from vendor A. It is also known that 20% of all those surveyed purchased their word-processing systems from vendor A. Given that a word processor was purchased from vendor A, what is the probability that the user is dissatisfied?

Sample space (S) = the users of word processors surveyed

Let \( W \) = the event that users of the word processor are dissatisfied with their word-processing system

\( X \) = the event that users bought their word processor from vendor A

\[
\begin{align*}
P(W) &= .10 \\
P(X) &= .20 \\
P(X \cap W) &= .05 \\
\end{align*}
\]

\[
\begin{align*}
P(W|X) &= \frac{P(X \cap W)}{P(X)} = \frac{.05}{.20} = \frac{1}{4} \\
\end{align*}
\]

Venn Diagram:

![Venn Diagram](image)
Example: Multiplicative rules of probability

A two-component electronic system is connected in parallel so that it fails only if both of its components fail. The probability that the first component fails is .10. If the first component fails, the probability that the second component fails is .05. What is the probability that the two-component electronic system fails?

Sol’n:

Let

\( C_1 = \) the event that the first component fails
\( C_2 = \) the event that the second component fails

\[ P(C_1) = .10 \]
\[ P(C_2|C_1) = .05 \]
\[ P(C_2 \cap C_1) = P(C_2|C_1)P(C_1) = .05(.1) = .005 \]
Example Problems: Probability

Example: Baye’s Rule

Example: Four students are selected at random from a civil engineering class and classified as male or female. List the elements of the sample space $S_1$ using the letter M for “male” and F for “female.” Define a second sample space, $S_2$, where the elements represent the number of females selected.
Solution:

$S_1 = \{\text{MMMM, MMMF, MMFM, MFMM, FMMM, MMFF, MFFM, FFMM, MFFF, FFFM, MFMF, FMMF, FMFM, FMFF, FFMF, FFFF}\}$

$S_2 = \{0, 1, 2, 3, 4\}$

Example: Let $R$ be the event that a red card is selected from an ordinary deck of 52 playing cards, and let $S$ be the entire deck. The $R'$ is the event that the card selected from the deck is not a red but a black card.


Example: Let $P$ be the event that a student selected at random while eating at the College canteen is a civil engineering student, and let $Q$ be the event that the person is a college junior. Then the event $P \cap Q$ is the set of junior civil engineering students eating at the College canteen.

Example: Let $M = \{a, e, i, o, u\}$ and $N = \{r, s, t\}$; then it follows that $M \cap N = \emptyset$. That is, $M$ and $N$ have no elements in common and, therefore, cannot both occur simultaneously.

Example: Let $A = \{a, b, c\}$ and $B = \{b, c, d, e\}$; then $A \cup B = \{a, b, c, d, e\}$.

Example: Combinatorial Methods

4/24) Students at a private liberal arts college are classified as being freshmen, sophomores, juniors, or seniors, and also according to whether they are male or female. Find the total number of possible classification for the students of this college.
Sol’n: $n_1*n_2 = 4*2 = 8$

7/24) A developer of a new subdivision offers a prospective home buyer a choice of 4 designs, 3 different heating systems, a garage or carport, and a patio or screened porch. How many different plans are available to this buyer?
Sol’n: $n_1*n_2*n_3*n_4 = 4*3*2*2 = 48$

10/24) In how many different ways can a true-false test consisting of 9 questions be answered?
Sol’n: $n_1*n_2*n_3*n_4*n_5*n_6*n_7*n_8*n_9 = 2*2*2*2*2*2*2*2*2 = 512$

14/24) (a) In how many ways can 6 people be lined up to get on a bus?
Soln: $n! = 6! = 720$
(b) If a certain 3 persons insist on following each other, how many ways are possible?

Soln: \( n_1! \times n_2! = 3! \times 4! = 144 \)

(c) If a certain 2 persons refuse to follow each other, how many ways are possible?

Soln: \( n \times n_1! \times n_2! = 10 \times 2! \times 4! = 480 \)

15/24) A contractor wishes to build 9 houses, each different in design. In how many ways can he place these houses on a street if 6 lots are on one side of the street and 3 lots are on the opposite side?

Soln: \( n! = 9! = 362,880 \)

17/24) In how many ways can 4 boys and 5 girls sit in a row if the boys and girls must alternate?

Soln: \( n_1! \times n_2! = 4! \times 5! = 2,880 \)

20/24) In how many ways can 5 starting positions on a basketball team be filled with 8 men who can play any of the position? Theorem 1.4

\[
\frac{n!}{(n-r)!} = \frac{8!}{(5-3)!} = 6720
\]

21/24) Find the number of ways in which 6 teachers can be assigned to 4 sections of an introductory psychology course if no teacher is assigned to more than one section.

\[
\frac{n!}{(n-r)!} = \frac{6!}{(6-4)!} = 360
\]

23/25) In how many ways can 5 different trees be planted in a circle?

Soln: \( (n-1)! = (5-1)! = 24 \)

26/25) In how many ways can 3 oaks, 4 pines, and 2 maples be arranged along a property line if one does not distinguish between trees of the same kind?

\[
\frac{n!}{n_1! \times n_2! \times n_3!} = \frac{9!}{3! \times 4! \times 2!} = 1,260
\]

27/25) A college plays 12 football games during a season. In how many ways can the team end the season with 7 wins, 3 losses, and 2 ties?

\[
\frac{n!}{n_1! \times n_2! \times n_3!} = \frac{12!}{7! \times 3! \times 2!} = 7,920
\]

28/25) Nine people are going on a skiing trip in 3 cars that will hold 2, 4, and 5 passengers, respectively. In how many ways is it possible to transport the 9 people to the ski lodge using all cars?
29/25) How many ways are there to select 3 candidates from 8 equally qualified recent graduates for openings in an engineering firm.

\[ \text{Soln: } nPr = \frac{8!}{3!(8-3)!} = 56 \]

Examples: (Additive Rules)

5/25) If A and B are mutually exclusive events and \(P(A) = 0.3\) and \(P(B) = 0.5\), find
(a) \(P(A \cup B)\); (b) \(P(A')\); (c) \(P(A' \cap B)\).

\[ \text{Soln:} \]
(a) \(P(A \cup B) = P(A) + P(B) = 0.3 + 0.5 = 0.8\)
(b) \(P(A') = 0.7\)
(c) \(P(A' \cap B) = 0.5\)

Example: Additive Rules

5/32) (Similar) The probability that a Japanese industry will locate in Bulacan is 0.7, the probability that it will locate in Laguna is 0.4, and the probability that it will locate in either Bulacan or Laguna or both is 0.8. What is the probability that the industry will locate
(a) in both province? (b) in neither province?

\[ \text{Soln:} \]
P(B) = 0.7, P(L) = 0.4, P(B \cup L) = 0.8.

(a) \[P(B \cup L) = P(B) + P(L) - P(B \cap L)\]
\[0.8 = 0.7 + 0.4 - P(B \cap L)\]
P(B \cap L) = 0.3
(b) \[P(B \cup L)' = 0.2\]

Venn diagram:
Example Problems: Probability

10/32) A pair of dice is tossed. Find the probability of getting (a) a total of 8; (b) at most a total of 5.

Soln:
(a) \( S = 6*6 = 36 \) (N=36). There are 5(n=5) possible combinations on the two dice to get a total of 8, therefore \( P = 5/36 \).
(b) There are 10(n=10) possible combinations on the two dice to produce at most a total of 5, therefore \( P = 10/36 = 5/18 \).

15/32) In a high school graduating class of 100 students, 54 studied mathematics, 69 studied history, and 35 studied both mathematics and history. If one of these students is selected at random, find the probability that

(a) the student took mathematics or history;
(b) the student do not take either of these subjects;
(c) the student took history but not mathematics.

Soln:
(a) \( P(A \cup B) = P(A) + P(B) - P(A \cap B) \)
\[
= \frac{54}{100} + \frac{69}{100} - \frac{35}{100} \\
= \frac{88}{100} = \frac{22}{25}
\]
(b) \( P(A \cup B)' = \frac{12}{100} = \frac{3}{25} \)
(c) \( P(B) = \frac{34}{100} = \frac{17}{50} \)

3/39) A random sample of 200 adults are classified below according to sex and the level of education attained.

<table>
<thead>
<tr>
<th>Education</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elementary</td>
<td>38</td>
<td>45</td>
</tr>
<tr>
<td>Secondary</td>
<td>28</td>
<td>50</td>
</tr>
<tr>
<td>College</td>
<td>22</td>
<td>17</td>
</tr>
<tr>
<td>Total</td>
<td>88</td>
<td>112</td>
</tr>
</tbody>
</table>

If a person is picked at random from this group, find the probability that
a) the person is a male, given that the person has a secondary education;
b) the person does not have a college degree, given that the person is a female.

Soln:
a) Let: \( M \) is the event that a male will be picked 
\( S \) is the event that the adult’s level of education is secondary education

<table>
<thead>
<tr>
<th>Education</th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elementary</td>
<td>38</td>
<td>45</td>
<td>83</td>
</tr>
<tr>
<td>Secondary</td>
<td>28</td>
<td>50</td>
<td>78</td>
</tr>
<tr>
<td>College</td>
<td>22</td>
<td>17</td>
<td>39</td>
</tr>
<tr>
<td>Total</td>
<td>88</td>
<td>112</td>
<td>200</td>
</tr>
</tbody>
</table>

\[
P(S \cap M) = \frac{28}{200} = \frac{28}{14} = \frac{2}{1}
\]

\[
P(M/S) = \frac{28}{200} = \frac{78}{200} = \frac{78}{39}
\]
Example Problems: Probability

b) Let \( F \) is the event that a female will be picked  
\( NC \) is the event that a person’s level of education is not a college degree  
\[ P(F \cap NC) = \frac{45+50}{200} = 0.95 \]  
\[ P(NC/F) = \frac{P(F \cap NC)}{P(F)} = \frac{95}{112} \]

Example: Conditional Probability

A class in CESTATS is comprised of 10 seniors, 30 sophomores, and 10 juniors. The final grades showed that 3 of the seniors, 10 of the sophomores, and 5 of the juniors received a grade of 4.0 for the subject. If a student is chosen at random from this class and is found to have earned a grade of 4.0, what is the probability that he or she is a sophomore?

Soln:

<table>
<thead>
<tr>
<th>Class</th>
<th>Grade</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.0</td>
<td>Not 4.0</td>
</tr>
<tr>
<td>Juniors</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Seniors</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Sophomores</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>Total</td>
<td>18</td>
<td>32</td>
</tr>
</tbody>
</table>

Let:  
- \( A \): event that a student earned a grade of 4.0  
- \( B \): event that a student is a sophomore

\[ P(A \cap B) = \frac{10}{50} \]  
\[ P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{10/50}{18/50} = \frac{5/9} {8/40} = 0.56 \]

The probability that an automobile being filled with gasoline will also need an oil change is 0.25; the probability that it needs a new oil filter is 0.40; and the probability that both the oil and filter need changing is 0.14.

a) If the oil had to be changed, what is the probability that a new oil filter is needed?  
b) If new oil filter is needed, what is the probability that the oil has to be changed?

Sol’n:

Let  
- \( A \): the event that the automobile needs to be filled with gasoline  
- \( B \): the event that the automobile needs an oil change  
- \( C \): the event that the automobile needs a new oil filter

Given: \( P(B|A) = 0.25, P(C) = 0.40, P(B \cap C) = 0.14 \)

\[ P(B \cap C) = P(B)P(C|B) \]  
Since gas filling is independent to oil change, \( P(B|A) = P(B) \)  
\[ 0.14 = (0.25) P(C|B) \]  
\[ P(C|B) = 0.56 \]

\[ P(C \cap B) = P(C)P(B|C) \]  
\[ 0.14 = 0.40 P(B|C) \]  
\[ P(B|C) = 0.35 \]
Example: Multiplicative rules

16/41) Two cards are drawn in succession from a deck without replacement. What is the probability that
a) both cards are red?
b) both cards are greater than 3 but less than 8?

Soln:

Defining the ff events:

A: the first card is red
B: the second card is red

a) \( P(A \cap B) = P(A)P(B|A) \)

\[
\begin{align*}
& = \frac{26}{52} \times \frac{25}{51} = \frac{650}{2652} = \frac{25}{102} \\
& = \frac{25}{102}
\end{align*}
\]

b) Defining the ff. events:

A: the first card is greater than 3 but less than 8
B: the second card is greater than 3 but less than 8

\[
\begin{align*}
P(AB) &= P(A)P(B|A) = \frac{16}{52} \times \frac{15}{51} = \frac{240}{2652} = \frac{20}{221} \\
&= \frac{20}{221}
\end{align*}
\]

1/45) In a certain region of the country it is known from past experience that the probability of selecting an adult over 40 years of age with cancer is 0.02. If the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the probability of incorrectly diagnosing a person without cancer as having the disease is 0.06, what is the probability that a person is diagnosed as having cancer?

Soln:

Consider the following events:

A: a person is diagnosed as having cancer
B1: a person has cancer
B2: a person without cancer

\[
= 0.02(0.78) + 0.98(0.06) \\
= 0.0744
\]

3/45) Referring to Exercise 1, what is the probability that a person diagnosed as having cancer actually has the disease?

\[
P(B1|A) = \frac{0.02(0.78)}{0.0744} = \frac{0.2097}{0.0744}
\]
Example Problems: Probability

2/45) Police plan to enforce speed limits by using radar taps at 4 different locations within city limits. The radar traps at each of the locations L1, L2, L3, and L4 are operated 40%, 30%, 20%, and 30% of the time, and if a person who is speeding on his way to work has probabilities of 0.2, 0.1, 0.5, and 0.2, respectively, of passing through these locations, what is the probability that he will receive a speeding ticket?

Soln:
Consider the following events:

A: the driver received a speeding ticket (caught when the radar is in operation)
B1: speeding driver passing through L1
B2: speeding driver passing through L2
B3: speeding driver passing through L3
B4: speeding driver passing through L4

Applying the rule of elimination,

\[ = 0.2 (0.4) + (0.1)(0.3) + (0.5)(0.2) + (0.3)(0.2) \]
\[ = 0.270 \]

4/45) If in Exercise 2 the person received a speed ticket on his way to work, what is the probability that he passed through the radar trap located at L2?
\[ = (0.1)(0.3)/(0.27) = 0.03/0.27 = 1/9 \]

Example: Baye’s Rule

5/45) Suppose that the four inspectors at a film factory are supposed to stamp the expiration date on each package of film at the end of the assembly line. John, who stamps 20% of the packages, fails to stamp the expiration date once in every 200 packages; Tom, who stamps 60% of the packages, fails to stamp the expiration date once in every 100 packages; Jeff, who stamps 15% of the packages, fails to stamp the expiration date once in every 90 packages; and Pat, who stamps 5% of the packages, fails to stamp the expiration date once in every 200 packages. If a customer complains that her package of film does not show the expiration date, what is the probability that it was inspected by John?

Consider the following events:

A: the product is not stamped
B1: the product is stamped by John
B2: the product is stamped by Tom
B3: the product is stamped by Jeff
B4: the product is stamped by Pat

Applying the rule of elimination, we can write

Example Problems: Probability

And the probabilities are
P(B_1)P(A|B_1) = (.20)(1/200) = .001
P(B_2)P(A|B_2) = (.60)(1/100) = .006
P(B_3)P(A|B_3) = (.15)(1/90) = .00167
P(B_4)P(A|B_4) = (.05)(1/200) = .00025

The probability that the product is not stamped is equal to
.001 + .006 + .00167 + .00025 = .00892

The probability that it was inspected by John is
=.001/.00892 = .1121
Example Problems: Probability

Example: Conditional probability
6/45) A regional telephone company operates three identical relay stations at different locations. During a one-year period, the number of malfunction reported by each station and the causes are shown below,

<table>
<thead>
<tr>
<th>Causes of Malfunction</th>
<th>Stations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
</tr>
<tr>
<td>Problems with electricity supplied (P1)</td>
<td>2</td>
</tr>
<tr>
<td>Computer malfunction (P2)</td>
<td>4</td>
</tr>
<tr>
<td>Malfunctioning electrical equipment (P3)</td>
<td>5</td>
</tr>
<tr>
<td>Caused by other human errors (P4)</td>
<td>7</td>
</tr>
</tbody>
</table>

Suppose that a malfunction was reported and it was found to be caused by other human errors. What is the probability that it came from station C?

Solution:
Consider the following events

Total no. of malfunction = A + B + C = 18 + 15 + 10 = 43

P(P4 \cap C) = 5/43

The malfunction was found to be caused by other human error

P(P4) = 19/43

The probability that it came from station C

\[
P(C|P4) = \frac{P(P4 \cap C)}{P(P4)} = \frac{5/43}{19/43} = \frac{5}{19} = 0.2632
\]